1996-97 School Year

Mathematics 30 Information Bulletin

Diploma Examinations Program

Students First!
Student Evaluation

Alberta
EDUCATION

This document was written primarily for:

Students	✓
Teachers	✓ Mathematics 30 teachers
Administrators	✓
Parents	
General Audience	
Others	

Distribution: Superintendents of Schools • School Principals and Teachers • The Alberta Teachers' Association • Alberta Education • General Public upon Request

Contents

Purpose of the Bulletin

The purpose of this bulletin is to provide students and teachers of Mathematics 30 with information about the diploma examinations scheduled for the 1996–97 school year.

We encourage teachers to share the contents of this bulletin with students and to review the scoring criteria with them.

If you have requests, questions, or comments about the contents of this bulletin, please contact:

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Important Dates in the 1996–97 School Year

Administration of the Examination

Alberta Education is increasing the number of administrations of diploma examinations to accommodate students' examination needs more effectively. For the 1996–97 school year the following diploma examinations will be offered in Mathematics 30.

1996–97 Administrations	*Time**
Friday, November 8	9:00–11:30 A.M.
Tuesday, January 28	9:00–11:30 A.M.
Wednesday, June 25	9:00–11:30 A.M.
Friday, August 15	9:00–11:30 A.M.

* The diploma examination is designed for a writing time of 2.5 h. Students may have an additional 0.5 h to complete the examination.

Please consult the *General Information Bulletin, Diploma Examinations Program*, for a complete schedule for all subject areas.

Scoring of the Examination

1996–97 Administrations	*Scoring Dates***
November	November 16
January	February 11 and 12 (Head Markers) February 13 to 15
June	July 5 and 6 (Head Markers) July 7 to 9
August	August 22

** Dates are tentative and will be confirmed by telephone with markers.

Notes of Interest

Information About Markers

The written-response questions of the Mathematics 30 diploma examinations are marked by Mathematics 30 teachers selected from those who have been recommended as markers to the Student Evaluation Branch by their superintendents. To qualify for recommendation, a teacher must have taught the complete Mathematics 30 course twice, be teaching the course in the current school year, and have an Alberta or Northwest Territories Permanent Professional Teaching Certificate.

Often, more teachers are recommended as markers by superintendents than are required by the Student Evaluation Branch for any one marking session. The following criteria are considered when markers are selected for a particular marking session:

- experience as a marker (experienced markers and first-time markers are included)
- regional representation (by zone, jurisdiction, and school)
- student population (proportional representation)

Teachers are particularly needed who can mark examinations written in French.

Teachers who wish to be recommended as markers for the diploma examinations should contact their superintendents by the following dates:

Diploma Date	Contact Superintendent by
January 1997	September 27, 1996
June & August 1997	February 28, 1997

Item Writing and Field Testing

Each school year, letters are sent to superintendents requesting their nominations for teachers to participate in field testing and item writing. **Teachers who are interested in these activities should let their superintendents know by September 12, 1996.** Teacher comments written on validation copies of Mathematics 30 field tests are used to assist the developers of the assessment material. Classroom teacher participation in the field test process is greatly appreciated.

As a result of teacher comments made on the validation copies of recent field tests, we would like to share some information.

- Some supervisors of field tests comment on the number of times that students inquire where to put the negative sign for numerical-response questions. It would be beneficial to your students to review the Mathematics 30 format before the students write the field tests and the diploma examination. The machine-scored numerical-response section in Mathematics 30 has no provision for negative signs.

- Teachers frequently ask why a question may ask for an answer to be recorded correct to the nearest tenth, when the correct result is a whole number. The reason students are asked to round is to avoid any confusion if they get an answer that is **not** a whole number.

- Some questions on the field tests expected students to derive $\sin(2\theta)$. Many teachers felt this was not in the curriculum. $\text{Sin}(2\theta)$ can be obtained from $\sin(A + B)$ as a problem-solving question.

Reports

After each January and June administration of the Mathematics 30 diploma examination, copies of the *Examiners' Report* for that particular administration are sent to teachers in all senior high schools in Alberta. The report contains provincial results along with commments about student performace on the examination. Examiners' reports are designed for teacher use. If we can make these reports more useful to you, please let us know.

School and Jurisdiction Statistical Reports are sent to each superintendent and senior high school principal after each January and June administration of the Mathematics 30 diploma examination. These reports provide detailed information on how well students in their school district and school, respectively, did on the Mathematics 30 diploma examination. Teachers may use these data to reflect on the areas of the program where their students did meet standards and those areas where their students did not meet the standards.

Each year, the Student Evaluation Branch produces the *Annual Report, Diploma Examinations Program.* This report contains information about the results achieved by all students who wrote diploma examinations in the preceding school year. It also contains special studies on topics of interest. The first annual report (1989–90 school year) contains a study comparing achievement in various diploma courses. The second report (1990–91 school year) contains a study comparing the achievement of students who repeat diploma examination courses

and rewrite diploma examinations with their achievement the first time and with the achievement of students who write only once. The third report (1991–92 school year) contains a study of participation rates in diploma examination courses. The fourth report (1992–93 school year) contains a study of English 30 students' application of conventions of language. The fifth report (1993–94 school year) presents a comparison of provincial participation rates in diploma examination subjects for the school years 1991–92, 1992–93, and 1993–94. The sixth report (1994–95 school year) examines declining enrollment of Grade 12 students. Each of these special studies will be of interest to teachers who are involved in interpreting diploma examinations results of their students.

Additional copies are available from the student Evaluation Branch. Phone: 427-0010 FAX: 403-422-4200
To call toll-free from outside of Edmonton, dial 310-000.

As of the 1995-96 school year, this report is available on the Internet at Alberta Education's Web Site (http://ednet.edu.gov.ab.ca).

Inservices and Presentations

On a limited basis and subject to budget constraints, Student Evaluation Branch staff is available to provide inservices or presentations related to diploma examinations or the interpretation of diploma examination results.

Student Use of Scientific Calculators

Examinations are constructed to ensure that the use of particular scientific calculators neither advantages nor disadvantages individual students.

Refer to Appendix A for the policy statement on the use of scientific calculators on diploma examinations. Students should be made aware of this policy **as early as possible** in the school term to ensure they are able to use the scientific calculator of their choice when writing examinations. **Students should know that notes stored in electronic devices may not be brought into the examination room.**

Students should also be made aware of the Examination Rules, Grade 12 Diploma Examinations in the *General Information Bulletin*.

Data Resources for Students

There is a tear-out formula sheet and a *z*-score page provided in all 1996–97 examinations.

Guide for Students

NEW

A publication written for students called *Students First: A Guide for Students Preparing to Write the Mathematics 30 Diploma Examination* is available in each senior high school. See your principal or department head for the black line master. This publication contains suggestions for exam preparation and exam writing suggestions. The Student Evaluation Branch highly recommends that students and teachers examine this document early in the term to prepare for the Mathematics 30 Examination. Parents will also find this to be a useful publication.

Student Requests for Rescores

Students may request a rescoring of their diploma examinations. Before applying for a rescoring, students should check their Diploma Examination Results Statement to find out the distribution of marks. The mark for the machine-scored questions is not likely to change as a result of a rescore, but the written-response mark could change slightly. Students should remember that the rescored mark will be the final mark whether it *increases* or *decreases*.

Students who decide to have an examination rescored must ensure that their application is received before the deadline specified on the results statement. The fee for rescoring each examination is $26.75, which includes GST. If a diploma examination mark is increased by 5% or more as a result of rescoring, the fee is refunded.

Student Requests for Rewriting

Students may rewrite a diploma examination in any regularly scheduled administration. Students must apply to the Student Evaluation Branch by September 11, 1996, November 15, 1996, and April 15, 1997, to be eligible to write the November, January, and June diploma examinations. The fee for rewriting each examination is $26.75, which includes GST. (For more details, see the *General Informaton Bulletin*.)

Mathematics 30 Information

Standards

Provincial standards help to communicate how well students need to perform to be judged as having achieved the learnings specified for Mathematics 30. According to the *Mathematics 30 Course of Studies*, student learnings refer to specific knowledge, skill, and attitude expectations. These learnings are amplified in Appendix E, Mathematics 30 Curriculum Standards, of this bulletin. Also included in Appendix E are examples of questions that students must be able to answer to demonstrate *acceptable* or *excellent* achievement. The examples provided are by no means exhaustive: they are intended to provide a profile of *acceptable* and *excellent* achievement.

Students who demonstrate *acceptable* achievement but not *excellent* achievement in Mathematics 30 will receive a final course mark between and including 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics but can anticipate difficulties if they choose to enroll in post-secondary mathematics courses. They have demonstrated mathematical skills and knowledge in the seven content strands of the Mathematics 30 curriculum and exhibit an ability to apply a broad range of problem-solving skills to these content strands.

Students who demonstrate *excellent* achievement will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and feel confident about their mathematical abilities. These students should encounter little difficulty in post-secondary mathematics programs; they should be encouraged to pursue careers in which they will use their talents in mathematics.

The specific statements of standards in Appendix E were written primarily to inform Mathematics 30 teachers about the extent to which students must know the Mathematics 30 content and must demonstrate the required skills to pass the examination.

Examination Design

The design of the Mathematics 30 diploma examinations in the 1996–97 school year is as follows:

Question Format	*Number of Questions*	*Percent Emphasis*
Multiple-Choice	40	57
Numerical-Response	9	13
Written-Response	3	30

The 1996–97 examinations consist of a multiple-choice and numerical-response section and a written-response section.

NEW

The numerical-response questions are dispersed throughout the multiple-choice questions, placed according to content topic.

Multiple Choice and Numerical Response

For multiple-choice questions, students are to choose the correct or best possible answer from four alternatives.

For numerical-response questions, students are to calculate a numerical answer. As well, students are to record their answer in a separate area of the answer sheet, usually correct to the nearest tenth or nearest hundredth. When the answer students are to record is not a decimal value (e.g., the number of people or the degree of a polynomial), students are asked to determine what "the number of people is ______" or what "the degree of this polynomial is ___________." If the answer can be a decimal value, then students are asked to record their answer correct to the nearest tenth or nearest hundredth.

Written Response

The written-response section focuses on students' understanding of the process of solving a problem and encourages students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. To achieve *excellence*, students must be able to select a strategy, carry it through, and complete the problem. The written-response section of the examination also focuses on students' understanding of mathematical concepts and allows for the most flexibility in gaining an understanding of students' communication and problem-solving abilities in mathematics.

Questions in the written-response section ask students to solve, explain their solution, justify their solution, or prove. For a definition of directing words that students may encounter on the diploma examination, see Appendix B.

In scoring the written-response section of the examinations, markers will evaluate how well students:

- understand the problem or the mathematical concept
- correctly use the mathematics
- use problem-solving strategies and explain their answer and procedures
- communicate their solutions and mathematical ideas

Above all, students should be encouraged to try to solve the problem. Even an attempt at a solution could be worth some marks. If students leave the paper blank, markers will not be able to award any marks. The three written-response questions are each scored on a five-mark scoring rubric. Samples of responses to written-response questions on the January 1996 examinations, and how they were scored, are presented in Appendix D.

Examination Specifications

Each Mathematics 30 Diploma Examination is designed to reflect the core content outlined in the *Mathematics 30 Course of Studies*. The examination is limited to those expectations that can be measured by a paper-and-pencil test. The time allotted to write the examination is two and one-half hours. Students may take an additional 0.5 h to complete the examination.

The content for the Mathematics 30 diploma examinations in the 1996–97 school year in the **machine-scored section** is emphasized as follows:

Content[1]	***Percent Emphasis***
Polynomial Functions	11
Trigonometric and Circular Functions	12
Statistics	5
Quadratic Relations	10
Exponential and Logarithmic Functions	11
Permutations and Combinations	10
Sequences and Series	11
Total Multiple-Choice and Numerical-Response	70
Written-Response	30

As published in the 1994–95 and the 1995–96 Mathematics 30 Diploma Examination information bulletins, the written-response questions will assess whether or not students can draw on their

[1]Content descriptions have been shortened in this table.

mathematical experiences to solve problems and to explain mathematical concepts. Therefore, the written-response questions will not necessarily fall into a particular unit of study but may cross more than one unit or may require students to make the connections between mathematical concepts.

Note: There are three written-response questions on the Mathematics 30 Diploma Examination. Each question is worth 10% of the entire examination. A sample of a general five-point scoring rubric is found in Appendix D of this bulletin.

The three mathematical understandings of procedures, concepts, and problem solving are addressed throughout the examination. Each understanding has the following emphasis:

Multiple-Choice and Numerical-Response	***Percent Emphasis***
Procedures	21
Concepts	24.5
Problem Solving	24.5
Written-Response	
Procedures, Concepts, Problem Solving	30

Each examination is built as closely as possible to these specifications.

Since the machine-scorable and written-response questions are weighted components, to calculate a student's examination mark, do the following calculation:

$$\left(\frac{x}{49} \times 70\right) + \left(\frac{y}{15} \times 30\right) = \text{exam mark}$$

x: Student total score on multiple-choice and numerical-response questions
y: Student total score on written-response question

Mathematics as Communication and Problem Solving

Communication

In keeping with the expectations listed in the *Mathematics 30 Course of Studies*, the 1996–97 examinations will reflect mathematics as communication. The program of studies includes communication in the problem-solving expectations: "Students will be expected . . . to read the problem thoroughly; identify and clarify key components; restate the problem, using familiar terms … ask relevant questions . . . document the solution process … and explain the solution in oral or written form …"
(From *Mathematics 30 Course of Studies*, pp. 6–7)

These expectations are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9 to 12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can
>
> - reflect upon and clarify their thinking about mathematical ideas and relationships
> - formulate mathematical definitions and express generalizations discovered through investigations
> - express mathematical ideas orally and in writing
> - read written presentations of mathematics with understanding
> - ask clarifying and extending questions related to mathematics they have read or heard about
> - appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas
>
> Focus: All students need extensive experience listening to, reading about, writing about, speaking about, reflecting on, and demonstrating mathematical ideas.
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 140)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as communication in the following manner:

> The assessment of students' ability to communicate mathematics should provide evidence that they can
>
> - express mathematical ideas by speaking, writing, demonstrating, and depicting them visually
> - understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms
> - use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 214)

Besides writing to communicate results, the accuracy of and logic in students' mathematical statements also reflects mathematics as communication. Hence, students will continue to be expected to demonstrate logical and meaningful communication on the diploma examinations.

Problem Solving

In keeping with the expectations identified in the *Mathematics 30 Course of Studies*, the 1996–97 examinations will reflect mathematics as problem solving. Problem solving is integrated throughout the content areas in the curriculum. A set of specific problem-solving learner expectations precedes the specific content learner expectations.

The expectations contained in the program of studies are consistent with the recommendations made by the National Council of Teachers of Mathematics:

> In grades 9 to 12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can
>
> - use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content
> - apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics
> - recognize and formulate problems from situations within and outside mathematics
> - apply the process of mathematical modeling to real-world problem situations
>
> Focus: In grades 9 to 12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the student's approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics as identified in later standards is both constructed and reinforced.
>
> (From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 137)

Assessment of Communication and Problem Solving in Mathematics

The open-ended question is a way in which to examine mathematics as communication and mathematics as problem solving. An open-ended question allows students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

Alberta Education recommends the following two documents, which examine the open-ended question in further detail:

> *Assessment Alternatives in Mathematics*: An overview of assessment techniques that promote learning
>
> *A Question of Thinking*: A First Look at Students' Performance on Open-ended Questions in Mathematics

Both these documents are available through:

> California State Department of Education
> Bureau of Publications, Sales Unit
> P. O. Box 271
> Sacramento, CA 95802-0271

The following document was recently published by the National Council of Teachers of Mathematics. It provides some practical suggestions for the assessment of problem solving and communication on a regular basis:

> *Mathematics Assessment*: Myths, Models, Good Questions, and Practical Suggestions

This document is available through:

> National Council of Teachers of Mathematics
> 1906 Association Drive
> Reston, VA 22091-1593

Appendix A

INSERT
CALCULATOR POLICY

Appendix B

Directing Words

Discuss

The word "discuss" will not be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity

The following words are specific in meaning.

Compare

Show the character or relative values of two things by pointing out their *similarities* and *differences*

Conclude

State a logical end based on reasoning and/or evidence

Contrast/Distinguish

Point out the *differences* between two things that have similar or comparable natures

Criticize

Point out the *merits* and *demerits* of an item or issue

Define

Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits

Describe

Give a written account or represent the characteristics of something by a figure, model, or picture

Design/Plan

Construct a plan, i.e, a detailed sequence of actions, for a specific purpose

Enumerate

Specify one by one or list in concise form and according to some order

Evaluate

Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages

Explain

Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail

How

Show in what manner or way, with what meaning

Hypothesize

Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically

Identify

Recognize and select as having the characteristics of something

Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram

Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence

Interpret
Tell the meaning of something; present information in a new form that adds meaning to the original data

Justify/Show How
Show reasons for or give facts that support a position

Outline
Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps

Predict
Tell in advance on the basis of empirical evidence and/or logic

Prove
Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons

Relate
Show logical or causal connection between things

Solve
Give a solution for a problem; i.e., explanation in words and/or numbers

Summarize
Give a brief account of the main points

Trace
Give a step-by-step description of the development

Why
Show the cause, reason, or purpose

Appendix C

Explanation of Mathematical Understandings

Procedures

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate
- give reasons for the steps in a procedure
- reliably and efficiently execute procedures
- verify the results of procedures empirically (e.g., using models) or analytically
- recognize correct and incorrect procedures
- generate new procedures and extend or modify familiar ones
- appreciate the nature and role of procedures in mathematics

It is important that students know how to execute mathematical procedures reliably and efficiently: a knowledge of procedures involves much more than simple execution. Students must know when to apply them, why they work, and how to verify that they have given a correct answer; they also must understand concepts underlying a procedure and the logic that justifies it. Procedural knowledge also involves the ability to differentiate those procedures that work from those that do not, and the ability to modify them or create new ones. Students must be encouraged to appreciate the nature and role of procedures in mathematics; that is, they should appreciate that procedures are created or generated as tools to meet specific needs in an efficient manner and thus can be extended or modified to fit new situations. The assessment of students' procedural knowledge, therefore, should not be limited to an evaluation of their facility in performing procedures: it should emphasize all the aspects of procedural knowledge addressed in this standard.

Concepts

The assessment of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts
- identify and generate examples and non-examples
- use models, diagrams, and symbols to represent concepts
- translate from one mode of representation to another
- recognize the various meanings and interpretations of concepts
- identify properties of a given concept and recognize conditions that determine a particular concept
- compare and contrast concepts

In addition, assessment should provide evidence of the extent to which students have integrated their knowledge of various concepts.

An understanding of mathematical concepts involves more than mere recall of definitions and recognition of common examples: it encompasses the broad range of abilities identified in this standard. Assessment, too, must address these aspects of conceptual understanding. Assessment tasks should focus on students' abilities to discriminate between the relevant and the irrelevant attributes of a concept in selecting examples and non-examples, to represent concepts in various ways, and to recognize students' various meanings. Tasks that ask students to apply information about a given concept in novel situations provide strong evidence of students' knowledge and understanding of that concept. Problems designed to elicit information about students' misconceptions can provide information useful in planning or modifying instruction.

Problem Solving

The assessment of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems
- apply a variety of strategies to solve problems
- solve problems
- verify and interpret results
- generalize solutions

Students' ability to solve problems develops over time as a result of extended instruction, opportunities to solve many kinds of problems, and encounters with real-world situations. Students' progress should be assessed systematically, deliberately, and continually to effectively influence their confidence and ability to solve problems in various contexts. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments should also yield evidence of students' use of strategies and problem-solving techniques and of their ability to verify and interpret results. Finally, because the power of mathematics is derived, in part, from its generalizability, this aspect of problem solving should be assessed as well.

From *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, p. 209, p. 223, p. 228.

Appendix D

Looking at Sample Responses

This section of the bulletin will provide some samples of responses to two of the three written-response questions from the January 1996 diploma examination.

Scoring Guide for Written-Response Questions

Credit may be given to students who show unusual insight. If their solutions fall outside *Specific Question Scoring Rubrics*, they will be scored against the *General Scoring Guide* shown below.

GENERAL SCORING GUIDE	
5 marks	The student • demonstrated a *complete understanding* of the problem • used mathematical knowledge and problem-solving techniques to find the solution • justified the solution and explained its relevance to the problem
4 marks	The student • demonstrated *an understanding* of the problem • chose a strategy that used mathematical knowledge and problem-solving techniques to find a solution, but the procedure contained *a minor flaw* • showed *some justification* of his or her results
3 marks	The student • demonstrated *some understanding* of the problem • formulated *some aspects* of the problem mathematically • demonstrated the use of a strategy that used mathematical knowledge and problem-solving techniques to find a *partial* solution • communicated little understanding of the complexities of the problem
2 marks	The student • explored the *initial stages* of the problem • applied *some* relevant mathematical knowledge and problem-solving techniques to find a *partial* solution
1 mark	The student • applied some relevant mathematical knowledge to the problem

SPECIFIC EXAMINATION QUESTIONS and SCORING GUIDES

1. In an arithmetic sequence, the first term is 84 and the common difference is –6. Find the two different values of n that satisfy $S_n = 612$. Explain why two different values for the number of terms can yield the same sum.

Question 1—Scoring Guide	
5 marks	The student • determined $n = 12$ and $n = 17$ • clearly explained why 2 different values for n can yield the same sum*
4 marks	The student • determined $n = 12$ and $n = 17$ • did not clearly explain why 2 different values for n can yield the same sum OR • attempted to find $n = 12$ or $n = 17$, but the strategy contained a minor error • clearly explained why 2 different values for n can yield the same sum*
3 marks	The student • determined $n = 12$ and $n = 17$ and linked the solution to **summation** OR • attempted to determine value(s) for n but did not complete the procedure, although a strategy was evident. Provided an explanation for why 2 values of n can yield the same sum*
2 marks	The student • determined $n = 12$ and $n = 17$ with supporting work • did not attempt to explain why 2 different values for n can yield the same **sum** OR • explained why 2 different values for n can yield the same sum but the explanation lacked clarity*
1 marks	The student • selected and applied a strategy that would have lead to the correct values for n if it had been carried out, e.g., wrote out the terms OR • substituted values from the question into an arithmetic sum formula OR • explained that 2 values of n can exist because the difference is negative

*Student does not have to use the arithmetic sequence given in the question to explain why two different value for n can yield the same sum.

Question 1

Sample 1

$$a = 84$$
$$d = -6$$
$$S_n = 612$$
$$612 = \frac{n(168 + (n-1)(-6))}{2}$$
$$0 = 84n - 3n^2 + 3n - 612$$
$$0 = 3n^2 - 87n + 612 = 3(n^2 - 29n + 204)$$

Solve for n using a calculator, quadratic formula, or factoring.

$$0 = 3(n-17)(n-12)$$
$$\therefore\ S_{12} = S_{17} = 612$$

Two different values can yield the same sum because it is possible that the sum of consecutive terms is zero.

In the example

$t_{13} + t_{14} + t_{15} + t_{16} + t_{17} = 0 \quad (12 + 6 + 0 + -6 + -12),$

we know $S_{12} + t_{13} + t_{14} + t_{15} + t_{16} + t_{17} = S_{17}$.

These two responses would receive a score of 5 because they demonstrate a complete understanding of the problem. These students determined $n = 12$ and $n = 17$ and clearly explained why two different values for the number of terms can yield the same sum.

Sample 2

$t_1 = 84 \qquad S_n = 612$

$d = -6$

$$S_n = \frac{n\,[2a + (n-1)d]}{2}$$
$$612 = n[168 + (n-1)-6]$$
$$= \frac{n\,[168 + {}^{-}6n + 6]}{2}$$
$$= \frac{174n - 6n^2}{2}$$
$$0 = -3n^2 + 87n - 612$$
$$= n^2 - 29n + 204$$
$$= (n-12)(n-17)$$
$$\boxed{n = 12 \text{ or } n = 17}$$

$S_{12} = \frac{12}{2}\,[168 + (11)-6] = \boxed{612} \qquad S_{17} = \frac{17}{2}\,[168 + (16)-6] = \boxed{612}$

The two different values for the number of terms can yield the same sum because the difference is a negative number. The terms for a sum of the first 12 are all positive, giving the answer 612. The terms between the 12th to 17th are {12, 6, 0, −6, −12}, the sum of these terms equals 0. Therefore when they are added to S_{12}, the sum stays the same.

Sample 3

$d = -6$
$t_1 = 84 = a$

$S_n = 612$

$612 = \frac{n}{2}(2a + (n-1)d)$
$612 = \frac{n}{2}(2(84) + (n-1)-6)$
$1224 = n(168 - 6n + 6)$
$1224 = 168n - 6n^2 + 6n$
$6n^2 - 174n + 1224 = 0$
$(n-12)(6n-102) = 0$

$n = 12$ or $6n = 102$
$n = 17$

They can have two different value of n. it is because both of them fix in the equation of $S_n = \frac{n}{2}(2a+(n-1)d)$.

This response would receive a score of 3. **It demonstrates an algebraic calculation of $n = 12$ and $n = 17$ and links the solution to summation in an attempt to explain why two different values for the number of terms can yield the same sum.**

Sample 4

a(first term) = 84
d (common difference = -6
Sn(Sum of 'n' terms) = 612
Because it is arithmetic we use the formula

$S_n = \frac{n(2a+(n-1)d)}{2}$

$612 = \frac{n[2\cdot(84) + (n-1)\cdot(6)]}{2}$

$1224 = n[168 + 6n - 6]$

$1224 = 168n + 6n^2 - 6n$ bring 1224 to the other side

$0 = 6n^2 + 162n - 1224$ they are all divisible by 6 therefore we take that out.

$0 = n^2 + 27n - 204$

This equation does not factor down perfectly therefore we must use the quadratic formula in order to find the value of n

the quadratic formula is $n = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$0 = n^2 + 27n - 204$
$\uparrow$ a, $\uparrow$ b, $\uparrow$ c

$n = \frac{-27 \pm \sqrt{(27)^2 - 4(1)(-204}}{2(1)}$

$n = \frac{-27 \pm \sqrt{1545}}{2}$

$n = 6.15$, $n = -33.15$

Two different values for the number of terms can yield the same sum because as they go through the formula they use different variables respectively and follow through with same answer.
for instance if you had the equation $x = \sqrt{16}$, it is possible for x to equal either +4 or -4.

This response would receive a score of 1. It demonstrates a selected and applied strategy that would have led to correct values of n if it had been carried out correctly. The attempted explanation regarding "Two different values for the number of terms can yield the same sum…," actually relates to the roots of a quadratic equation and not to the summation of a sequence, as required.

2. Algebraically show why the **only** solutions to the equation $\log_x(19x - 30) = 3$ are $x = 2$ and $x = 3$.

Question 2—Scoring Guide	
5 marks	The student • demonstrated a correct algebraic procedure or explanation to solve $\log_x (19x - 30) = 3$ • explained why $x = -5$ is not part of the solution
4 marks	The student • demonstrated a correct algebraic procedure or explanation to solve $\log_x (19x - 30) = 3$ AND • gave a weak explanation why $x = -5$ is not part of the solution
3 marks	The student • demonstrated a correct algebraic procedure or explanation to find the solutions to $0 = x^3 - 19x + 30$ as being 2, 3, –5 OR • found the solution to $\log_x (19x - 30) = 3$ with no procedure shown and explained why $x = -5$ is not part of the solution
2 marks	The student • found the solutions of $\log_x (19x - 30) = 3$ as 2, 3, –5 with no procedure shown OR • demonstrated a correct algebraic procedure or explanation to find the solutions to $0 = x^3 - 19x + 30$ but made an error
1 marks	The student • started a procedure, e.g., changed the equation to exponential form OR • verified by substitution that $x = 2$ and $x = 3$ satisfy the equation $\log_x (19x - 30) = 3$. This is not sufficient to show why the solution consists of only 2 values for x

Question 2

Sample 1

$$\log_x (19x - 30) = 3$$
$$x^3 = 19x - 30$$
$$x^3 - 19x + 30 = 0$$
$$\text{let} \quad P(x) = x^3 - 19x + 30$$

Since $P(x)$ is a third-degree polynomial, there will be at most 3 values for x that satisfy $x^3 - 9x + 30 = 0$.

$$P(3) = 0 \text{ so } (x-3) \text{ is a factor of } P(x)$$
$$P(x) = (x-3)(x^2 + 3x - 10)$$
$$P(x) = (x-3)(x+5)(x-2)$$

The solution of $P(x) = 0$ is 3, –5 and 2.

The solution of $\log_x (19x - 30) = 3$ is $x = 3$ and $x = 2$, since $\log_3 27 = 3$ and $\log_2 8 = 3$.

Since $\log_b x$ must have $x > 0$, $x = -5$ is not a solution of $\log_x (19x - 30) = 3$ because $19(-5) - 30$ is less than zero.

OR

The base of a logarithm must be positive, hence $x > 0$ and $x = -5$ is not a solution.

These two responses would receive a score of 5 because they demonstrate a correct algebraic procedure or explanation to solve $\log_x (19x - 30) = 3$. In addition, the responses explain why $x = -5$ is not part of the solution.

Sample 2

$\log_x(19x-30)=3$ Change to an exponential form

$x^3 = 19x - 30$

$x^3 - 19x + 30 = 0$ Divide by $x-2$

$$\begin{array}{r} x^2+2x-15 \\ x-2\overline{)x^3+0x^2-19x+30} \\ \underline{-x^3+2x^2} \\ 2x^2-19x \\ \underline{-2x^2+4x} \\ -15x+30 \\ \underline{-15x+30} \\ 0 \end{array}$$

Divide the quotient x^2+2x by $x-3$

$$\begin{array}{r} x+5 \\ x-3\overline{)x^2+2x-15} \\ \underline{-x^2+3x} \\ 5x-15 \\ \underline{5x-15} \\ 0 \end{array}$$

The third factor is, $x+5$

The third factor is not a valid answer though. Check by substituting it into orginal formula.

$\log_{(-5)}(19(-5)-30)=3$

$\dfrac{\log_{10}(-95-30)}{\log_{10}(-5)} \neq 3$

X cannot be –5 becouse it gives you a negative base which is unsolvable. Plus the value that is being logged is negative, which is also unsolvable. therefore the only solutions are $x=2$ and $x=3$

Sample 3

$$\log_x (19x-30) = 3$$
$$x^3 = 19x-30$$
$$x^3 - 19x + 30 = 0$$
$$(x-3)(x-2)(x^2+5) = 0$$
$$x = 3, 2, \sqrt{-5}$$

$$\begin{array}{r|rrr|r} 3 & 1 & 0 & -19 & 30 \\ & & & 9 & -30 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$$\begin{array}{r|rr|r} 2 & 1 & 3 & -10 \\ & & 2 & 10 \\ \hline & 1 & 5 & 0 \end{array}$$

This response would receive a score of 2. It demonstrates a correct algebraic procedure or explanation to find the solution to $0 = x^3 - 19x + 30$, but contains an error.

Sample 4

$\log_x (19x-30) = 3 \quad x = 2 \;\&\; x = 3$

$$\log_2 (19(2)-30) = 3$$
$$\log_2 (38-30) = 3$$
$$\log_2 8 = 3$$
$$\log_2 8 = \log_2 2^3$$
$$\log_2 8 = \log_2 8$$
$$8 = 8 \leftarrow \text{equal each other}$$

$$\log_3 (19(3)-30) = 3$$
$$\log_3 (57-30) = 3$$
$$\log_3 27 = 3$$
$$\log_3 27 = \log_3 3^3$$
$$\log_3 27 = \log_3 27$$
$$27 = 27 \leftarrow \text{equal each other}$$

This response would receive a score of 1. The response verifies by substitution that $x = 2$ and $x = 3$ satisfy the equation $\log_x (19x - 30) = 3$. This is not sufficient to show why the solution consists of only 2 values for x.

Appendix E

Mathematics 30 Curriculum Standards

Mathematics 30 Curriculum Standards

The Curriculum Standards provided in this appendix are intended to clarify the Mathematics 30 Course of Studies *statements. Included are examples of questions that students must be able to do to demonstrate* acceptable *or* excellent *achievement. For a definition of* acceptable *and* excellent *achievement, see page 7 of this bulletin.*

Problem Solving

Students in Mathematics 30 can participate in and contribute toward the problem-solving process for problems within the seven content strands. [1]

Examples

- Given the solution to a problem, analyze the solution for correctness, provide the correct response, and provide possible reasons for errors. For example:

  Menghsha examined the graph of the function $y = 3\sin\theta$ and determined that the domain of the function was $-1 \le \theta \le 1$. Is Menghsha's answer correct? If not, provide the correct answer and explain Menghsha's error.

- Given one method of solving a problem, solve it a second way. For example:

  Jillian was asked to find the factors of $P(x) = x^3 - 9x^2 - x + 9$. On her graphing calculator, she graphed the function and determined that the factors for $P(x)$ were $(x - 9)$, $(x + 1)$, and $(x - 1)$. If Jillian was unable to graph $P(x)$, show another method that Jillian could have used to find its factors.

- Given that a ferris wheel with a radius of 18 m makes a complete revolution in 12 s, draw a diagram of the situation and create a table of values showing the relationship between the height h of a rider above the ground (the lowest point of the ferris wheel is 1 m above the ground) and the time t to determine the height of a rider after 6 s.

- Given a problem, solve it for the specific case(s) and then provide a general solution. For example:

  Given a 4-sided and a 10-sided polygon, determine the number of diagonals in each. Also determine a general statement about the number of diagonals in an n-sided polygon.

- Given that a ferris wheel with a radius of 18 m makes a complete revolution in 12 s, develop a mathematical model that describes the relationship between the height h of a rider above the lowest point of the ferris wheel, which is 1 m above the ground, and the time t. Provide a full explanation of how your model was developed and suggest alternative ways of developing the model.

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[1] Italicized comments give an overview of the curriculum statements to be found in the *Mathematics 30 Course of Studies.*

Polynomial Functions

Given any integral polynomial function of degree 3 or less, students can determine its zeros, its factors, and its graph, and can describe, in writing, the relationship among its zeros, its factors, and its graphs.

Acceptable Standard

The student can

- recognize and give examples of polynomial functions of different degrees
- generate the graph of any integral polynomial function with the use of graphing calculators or graphing utility packages
- use the Remainder Theorem to evaluate a third-degree integral polynomial function for rational values of the variable and to understand how this can be used to find factors of the polynomial function
- factor and find the zeros for an integral polynomial function in standard form, degree 3 or less, in which all zeros are rational
- using graphing calculators or computers, find approximations for all the real zeros of integral polynomial functions
- derive an equation of an integral third-degree polynomial function, given its rational zeros

Excellent Standard

The student can also

- use the Remainder Theorem when either the factor or the original polynomial contains unknown coefficients
- use the Remainder Theorem to evaluate integral polynomial function beyond the third degree for rational values of the variable, and understand how this can be used to find factors of the polynomial function
- derive an equation for an integral polynomial function, given its zeros and any other information that will uniquely define it

- recognize the general shape of graphs of integral polynomial functions of degree 4 or less where the multiplicity of zeros is one, two, or three
- identify the potential rational zeros of an integral polynomial function
- determine the minimum degree of a polynomial function by using the multiplicities of its zeros
- participate in and contribute toward the problem-solving process for problems that can be represented by polynomial functions studied in Mathematics 30

- recognize the general shape of graphs of integral polynomial functions of degree n where the multiplicity of zeros is greater than two
- explain the relationships between the graphs of different polynomial functions and their zeros
- complete the solution to problems that can be represented by polynomial functions studied in Mathematics 30

Examples

- Given $P(x) = 10x^3 + 51x^2 + 3x - 10$, determine its zeros, its factors, and its graph. The student can also describe the relationship among its zeros, its factors, and its graph

- Given $P(x) = ax^3 + bx^2 + 3$ and that the remainders are 7 and 10 when divided by $x - 2$ and $x + 1$ respectively, determine the zeros, the factors, and the graph of $P(x)$

Trigonometric and Circular Functions

Students should be able to solve a first-degree equation that involves the primary trigonometric functions and describe the relationship between its root(s) and the graph of its corresponding function.

Students can also demonstrate, by simplifying and evaluating trigonometric expressions, an understanding that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

Acceptable Standard The student can	*Excellent Standard* The student can also
• convert angle measurements between degree and radian measure	
• given any two of the following measurements—the radian measure of the central angle, the radius, or the length of an arc—determine the unknown measurement	
• verify[2] the fundamental trigonometric identities	• prove[3] trigonometric identities
• solve first-degree trigonometric equations on the domain $0 \leq \theta < 2\pi$ in radians and $0° \leq \theta < 360°$	• solve first- and second-degree trigonometric equations, including double and half angles on the domain $0 \leq \theta < 2\pi$ and $0° \leq \theta < 360°$
• simplify and evaluate simple trigonometric expressions involving the fundamental trigonometric identities	
• generate the graph of trigonometric functions with the use of graphing calculators or graphing utility packages	

[2]For a definition of **verify**, see the *Mathematics 30 Course of Studies*, p. 6.

[3]For a definition of **prove,** see the *Mathematics 30 Course of Studies*, p. 6.

- explain the effect of each parameter a, b, c, and d on the graph of the $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$ functions

- state the domain and range of $y = \sin\theta$, $y = \cos\theta$, and $y = \tan\theta$

- describe, orally and in writing, the relationship between the root(s) of a first-degree trigonometric equation and the graph of its corresponding function

- participate in and contribute toward the problem-solving process for problems that can be represented by trigonometric functions studied in Mathematics 30

- explain, orally and in writing, the combined effects of the parameters a, b, c, and d in the trigonometric functions $y = a \sin[b(\theta + c)] + d$ and $y = a \cos[b(\theta + c)] + d$, on the functions' domain and range

- describe, orally and in writing, the relationship between the root(s) of a trigonometric equation and the graph of its corresponding function

- complete the solution to problems that can be represented by trigonometric functions studied in Mathematics 30

Examples

- Given $y = 2\sin\left(\theta - \frac{1}{2}\right)$, $0 \le \theta < 2\pi$, determine its zeros, describe orally and in writing the relationship between its zeros and its corresponding graph, and describe the effect that 2 and $-\frac{1}{2}$ have on the graph of $y = \sin\theta$.

- Given $\frac{\cot\theta}{\tan\theta}$, use fundamental trigonometric identities to simplify the expression and verify this simplification by substituting values for the variable and by comparing their corresponding graphs.

- Given $2 - 2\cos^2\theta = \sin\theta$, $0 \le \theta < 2\pi$, determine its zeros and describe, orally and in writing, the relationship between its zeros and the graphs of $y = 2 - 2\cos^2\theta$ and $y = \sin\theta$.

Statistics

Students can describe and analyze data by using the characteristics of a normal distribution.

Acceptable Standard	*Excellent Standard*
The student can	The student can also
• describe the characteristics of normally distributed data	
• interpret the mean and standard deviation of a set of normally distributed data	
• apply the standard normal curve and the z-scores of data that are normally distributed	
• participate in and contribute toward the problem-solving process for problems that require the analysis of statistics studied in Mathematics 30	• complete the solution to problems that require the analysis of statistics studied in Mathematics 30

Examples

• Given that the results of a test were normally distributed with a mean of 30 and a standard deviation of 5, and given that the passing mark was set at 25, determine the percentage of students who passed the test	• Given that the results of a test were normally distributed with a mean of 30 and a standard deviation of 5, and given that the passing mark was set at 25, determine the percentage of students who passed the test.

Quadratic Relations

Students can describe the conditions that generate quadratic relations.

Acceptable Standard The student can	*Excellent Standard* The student can also
• describe orally, in writing, and by modelling, the intersection of a plane and a conical surface that would result in a hyperbola, an ellipse, a parabola, and a circle	
• describe orally, in writing, and by modelling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate ellipse and hyperbola	• describe orally, in writing, and by modelling, and identify the position of the plane at which the intersection of a plane and a conical surface defines a degenerate parabola
• generate the graph of quadratic relations with the use of graphing calculators or graphing utility packages	
• describe, orally and in writing, and identify the quadratic relation defined by a combination of numerical coefficients for any quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$	
• describe, orally and in writing, and identify the quadratic relation formed when given the value of the eccentricity	
• describe, orally and in writing, and identify the eccentricity when given the quadratic relation	• describe, orally and in writing, the combination of values for the numerical coefficients of the general quadratic relation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, that would result in the degenerate conics
• describe, orally and in writing, and identify the quadratic relation formed when given the locus definition	• describe, orally and in writing, and identify the changes in the graph of a quadratic relation when the eccentricity changes

- identify and graph the quadratic relation when given a point on the quadratic relation, a fixed point, and the eccentricity

- calculate the eccentricity when given a fixed horizontal or vertical line, a fixed point, and a point on the quadratic relation

- identify and graph the quadratic relation when given the eccentricity, a fixed point, and a fixed horizontal or vertical line

- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when **two** of the numerical coefficients change

- participate in and contribute toward the problem-solving process for problems that require the analysis of quadratic relations studied in Mathematics 30

- use the locus definition to verify the equation of each conic section

- describe, orally and in writing, and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change

- complete the solution to problems that require the analysis of quadratic relations studied in Mathematics 30

Examples

- Given $2x^2 + 2y^2 + x - 3y - 25 = 0$, identify the quadratic relation described by this equation.
- Given $Ax^2 + Cy^2 + Dx - Ey - 36 = 0$, identify this as a hyperbola when $AC < 0$.
- Given that a quadratic relation is represented by
  $3x^2 + 4y^2 + 5x + Ey - 36 = 0$, where $B = 0$, describe orally or in writing what happens to the graph of this quadratic relation when 5 is changed to –4 and –36 is changed to –9.
- Given a quadratic relation that is described as having an eccentricity of 2, identify this as a hyperbola and describe its locus.
- Given that the locus of points such that the sum of the distances between one of the points and two fixed points is constant, identify this locus as an ellipse.
- Given a description of the intersection of a plane and a conical surface, identify the conic section formed.
- Given that the cutting plane approaches the vertex of the conical surface, describe orally, in writing, and by modelling, the effect on the ellipse.
- Given that the eccentricity of the orbit of Halley's comet, which has a period of 76 years, is 0.96, sketch its graph.
- Describe and identify the effects on the graph of the quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, when two or more of the numerical coefficients change.
- Given a description of the intersection of a plane and a conical surface, identify orally or in writing the degenerate parabola formed.
- Given the equation of a degenerate quadratic relation in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, describe orally and in writing the quadratic relation formed.
- Given the graph and the eccentricity of a quadratic relation, describe orally and in writing the changes to the graph when the eccentricity changes.
- Given that the fixed point of an ellipse is moving closer to the centre of the ellipse, describe orally and in writing the effect on the eccentricity.
- Given that the eccentricity of any quadratic relation is the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line, describe orally and in writing the effect that changing the eccentricity has on the relative positions of the fixed line and the fixed point.

Exponential and Logarithmic Functions

Students can describe the relationship between exponential and logarithmic functions.

Acceptable Standard

The student can

- generate the graph of exponential and logarithmic functions with the use of graphing calculators or graphing utility packages
- recognize and sketch the graphs of exponential and logarithmic functions and recognize their inverse relationship
- convert functions from exponential form to logarithmic form and vice versa
- apply the laws and properties of logarithms to evaluate logarithmic expressions
- solve and verify simple exponential and logarithmic equations
- state the domain and range of the exponential and logarithmic functions
- use the graphs of the exponential and logarithmic functions to estimate the value of one of the variables, given the other variable
- participate in and contribute toward the problem-solving process for problems that can be represented by logarithmic or exponential functions studied in Mathematics 30

Excellent Standard

The student can also

- solve and verify exponential and logarithmic equations
- complete the solution to problems that can be represented by logarithmic or exponential functions studied in Mathematics 30

Examples

- Given $f(x) = 4^{2x}$, sketch its graph, describe its domain and range, find the zeros of its corresponding equation, and describe orally and in writing the relationship between the roots of its equation and its graph. The student can write the inverse of $f(x) = 4^{2x}$ in logarithmic form, sketch its graph, describe its domain and range, determine the zeros of this equation, and describe orally and in writing the relationship between the zeros of its equation and its graph.

- Given the equation $\log_5(x - 4) + \log_5(x - 2) = 3$, find all the possible values of x, identify the domain, describe orally and in writing the relationship between the roots of this equation and its graph, and describe orally and in writing the reasons why there are values of x that satisfy the equation but are not permissible for the function.

Permutations and Combinations

*Students can describe the difference between a permutation and a combination, calculate the number of permutations or combinations of **n** things taken **r** at a time, and apply these to the expansion of binomials.*

Acceptable Standard

The student can

- calculate the number of linear, circle, and ring permutations and permutations with repetitions of n things taken r at a time
- calculate the number of combinations of n things taken r at a time
- expand binomials of the form $(x + a)^n$, $n \in W$, using the Binomial Theorem
- describe, orally and in writing, the difference between a permutation and a combination
- participate in and contribute toward the problem-solving process for problems involving permutations and/or combinations, including probability problems, studied in Mathematics 30

Excellent Standard

The student can also

- explain the reason that there are different numbers of permutations when a given number of objects are arranged in a line, a circle, or a ring, or when some of the objects are repeated or identical
- expand binomials of the form $(x + by)^n$, $n \in W$, using the binomial theorem, and determine specific terms of this expansion
- complete the solution to problems involving permutations and/or combinations, including probability problems, studied in Mathematics 30

Examples

- Given that there are 10 musicians in the finals of a music competition, decide whether permutations or combinations should be used to calculate the number of ways in which first, second, and third prizes can be awarded.
- Given the binomial $(x + 2)^5$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $x^4(2)$ is obtained, determine the number of terms in the expansion of $(x + 2)^5$, and describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial.
- Given that five people can sit at a round table, decide whether permutations or combinations should be used to determine the number of different orders in which these five can sit at the table if Jack and Jill must sit next to one another. The student can also justify the method of solution.
- Given the binomial $(3x - 2y)^7$, find the coefficient of the x^3 term in the expansion, determine how the coefficient of the term containing $[(3x)^6(2y)]$ is obtained, and determine the number of terms in the expansion of $(3x - 2y)^7$. The student can describe orally and in writing the relationship between the number of terms in the expansion and the exponent of the binomial, how the number of a given term in the expansion of $(3x - 2y)^7$ relates to the exponent of $(2y)$ in that term, and how the coefficients of the terms that are equidistant from the ends of the expansion of $(3x - 2y)^7$ compare in terms of combinations.

Sequences and Series

Students can describe the differences between sequences and series with an emphasis on arithmetic and geometric sequences, can determine the terms of arithmetic and geometric sequences, and can determine the sums of arithmetic and geometric series.

Acceptable Standard The student can	*Excellent Standard* The student can also
• write the specific terms of a sequence, given its defining function	• write the specific terms of a sequence when given its recursive definition • determine the functions describing any sequence that has a recognizable pattern
• expand a series given in sigma notation	
• describe, orally and in writing, the difference between sequences and series, arithmetic or geometric, infinite and finite	
• apply the general term "formula" to arithmetic and geometric sequences	• solve problems using the general term and/or sum formulas in which there are two unknowns
• apply the sum formula for arithmetic and geometric series	
• participate in and contribute toward the problem-solving process for problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30	• complete the solution to problems involving sum and term formulas for arithmetic and geometric series and sequences studied in Mathematics 30

Examples

- Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, and determine the sum of a specified number of terms.
- Given that a series is defined by $\sum_{n=3}^{6}(-2)^n$, write the terms of the series, determine whether the series is arithmetic or geometric, and determine the sum of the series.

- Given that a sequence is defined by $t_n = 5n - 3$, write the terms of the sequence, determine whether the sequence is arithmetic or geometric, determine the common difference or common ratio, determine the related series, determine the sum of a specified number of terms, and determine the formula for the sum of n terms.
- Given an arithmetic sequence where $t_4 + t_{13} = 99$ and $t_7 = 39$, determine the first term of this sequence.

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